Math Circle:
Perfect Numbers

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The Greek letter σ is called sigma.
If n is an integer

$$\sigma(n) = \text{ sum of the divisors of } n.$$

Examples:
$\sigma(1) = 1$,
$\sigma(2) = 1 + 2 = 3$,
$\sigma(3) = 1 + 3 = 4$,
$\sigma(4) = 1 + 2 + 4 = 7$,
$\sigma(5) = 1 + 5 = 6$,
$\sigma(6) = 1 + 2 + 3 + 6 = 12$.

1. Compute:

$\sigma(1) =$
$\sigma(2) =$
$\sigma(3) =$
$\sigma(4) =$
$\sigma(5) =$
$\sigma(6) =$
$\sigma(7) =$
$\sigma(8) =$
$\sigma(9) =$
$\sigma(10) =$
$\sigma(11) =$
$\sigma(12) =$
$\sigma(13) =$
$\sigma(14) =$
$\sigma(15) =$
$\sigma(16) =$
$\sigma(17) =$
$\sigma(18) =$
$\sigma(19) =$
$\sigma(20) =$
$\sigma(21) =$
$\sigma(22) =$
$\sigma(23) =$
$\sigma(24) =$
$\sigma(25) =$
$\sigma(26) =$
$\sigma(27) =$
$\sigma(28) =$
$\sigma(29) =$
$\sigma(30) =$
$\sigma(31) =$
$\sigma(32) =$

2. A number n is called perfect if $\sigma(n) = 2n$.
Example: $\sigma(6) = 12$. We have that 6 is a perfect number.
Find more perfect numbers.

3. Find numbers n so that $\sigma(n) > 2n$.

4. If p is a prime number, then what is $\sigma(p)$? What are $\sigma(p^2)$, $\sigma(p^3)$?

$\sigma(2) =$	$\sigma(2^2) =$	$\sigma(2^3) =$
$\sigma(3) =$	$\sigma(3^2) =$	$\sigma(3^3) =$
$\sigma(5) =$	$\sigma(5^2) =$	$\sigma(5^3) =$
$\sigma(7) =$	$\sigma(7^2) =$	$\sigma(7^3) =$
$\sigma(11) =$	$\sigma(11^2) =$	$\sigma(11^3) =$
$\sigma(13) =$	$\sigma(13^2) =$	
$\sigma(17) =$	$\sigma(17^2) =$	
$\sigma(19) =$	$\sigma(19^2) =$	
$\sigma(23) =$	$\sigma(23^2) =$	
$\sigma(29) =$	$\sigma(29^2) =$	
$\sigma(31) =$	$\sigma(31^2) =$	

5. Compute σ(powers of 2).

$\sigma(2^1) =$
$\sigma(2^2) =$
$\sigma(2^3) =$
$\sigma(2^4) =$
$\sigma(2^5) =$
$\sigma(2^6) =$
$\sigma(2^7) =$
$\sigma(2^8) =$

6. Compare $\sigma(a)\sigma(b)$ to $\sigma(ab)$:

$\sigma(2)\sigma(3) =$	$\sigma(6) =$
$\sigma(2)\sigma(5) =$	$\sigma(10) =$
$\sigma(2)\sigma(7) =$	$\sigma(14) =$
$\sigma(2)\sigma(13) =$	$\sigma(26) =$
$\sigma(4)\sigma(3) =$	$\sigma(12) =$
$\sigma(4)\sigma(5) =$	$\sigma(20) =$

$\sigma(2)\sigma(2) =$	$\sigma(4) =$
$\sigma(2)\sigma(4) =$	$\sigma(8) =$
$\sigma(2)\sigma(6) =$	$\sigma(12) =$
$\sigma(4)\sigma(6) =$	$\sigma(24) =$
$\sigma(3)\sigma(3) =$	$\sigma(9) =$
$\sigma(3)\sigma(9) =$	$\sigma(27) =$
$\sigma(3)\sigma(6) =$	$\sigma(18) =$

Homework: 1. Find more perfect numbers ($\sigma(n) = 2n$).
2. Give more examples of numbers n such that $\sigma(n) > 2n$.
3. Give examples of numbers n such that $\sigma(n) > 3n$.
4. Is it true that $\sigma(n)$ is always less than $4n$?