Monopoly

Question: Which properties are the best?
This is open-ended. We won't end up answering this question, but it's a good starting point.

There are many different ways you could go with this. We will eventually focus on one, but first let's brainstorm.

( - - - )

List factors:
( - - - )

If we accounted for all of these, then the answer would depend on the situation, and we'd really be trying to answer "what should you do at any given time?"
Value of a property:
Balance how much it costs, how much $ you get when people land on it, and how much people land on it.
Let's focus on $0.5 factor... so, this is a talk about applying some specific ideas from probability.
So: Which squares are most likely to be landed on?

(......)

Complication we want to ignore: Everyone starts in one spot.

What we really want to answer: If a player has been moving for a while, then what is the prob. we'll find him in each particular square?
Let's try this for a much simpler game board:

A

B

C

D

Your "die": 1/2 prob. moving 1
1/2 prob. moving 1

and if you land on C, you automatically go to A.

A ---- 1/2 B

A ---- 1/2 C

B ---- 1/2 C

B ---- 1/2 D

C ---- 1/2 A

C ---- 1/2 D

2 ways

D ---- 1/2 A

D ---- 1/2 B

2 ways

solve exactly

compute
Compute

Say we start at A, we see what happens:

1. $A \rightarrow \frac{1}{8} B$ \quad $\rightarrow$ \quad $\frac{1}{2}(\frac{1}{8} C + \frac{1}{8} D)$ \quad $\rightarrow$ \quad $\frac{1}{4}(\frac{1}{8} A) + \frac{1}{4}(\frac{1}{8} B)$

\[
\begin{align*}
A & = \frac{3}{8} A \\
B & = \frac{3}{8} B \\
C & = \frac{3}{8} C \\
D & = \frac{3}{8} D
\end{align*}
\]

\[
\begin{align*}
\frac{1}{4}(\frac{1}{8} B + \frac{1}{8} C) & \quad \rightarrow \quad \frac{1}{4}(\frac{1}{8} B + \frac{1}{8} C) \\
\frac{1}{4}(\frac{1}{8} C + \frac{1}{8} D) & \quad \rightarrow \quad \frac{3}{16}(\frac{1}{8} C + \frac{1}{8} D) \\
\frac{3}{8}(\frac{1}{8} A + \frac{1}{8} D) & \quad \rightarrow \quad \frac{3}{8}(\frac{1}{8} A + \frac{1}{8} B)
\end{align*}
\]

\[
\begin{align*}
A & = \frac{15}{32} A \\
B & = \frac{7}{32} B \\
C & = \frac{7}{32} C \\
D & = \frac{7}{32} D
\end{align*}
\]

\[
\begin{array}{c}
\text{Is there another way?}
\end{array}
\]

\[
\begin{array}{c}
\text{3529} \\
\text{2353} \\
\text{2941} \\
\text{1176}
\end{array}
\]
Well, if we found the right probabilities, then if we start with that mix of probabilities and move the system forward one step, the mix should stay the same.

[Justify this.]

So, we want \( p_A, p_B, p_C, p_D \) s.t.

\[
egin{align*}
    p_A (\frac{1}{2} B + \frac{1}{3} C) &= (p_C + \frac{1}{2} p_D) A = p_A A \\
    p_B (\frac{1}{3} C + \frac{1}{2} D) &= (\frac{1}{3} p_A + \frac{1}{4} p_D) B = p_B B \\
    p_C (A) &= (\frac{1}{4} p_A + \frac{1}{2} p_B) C = p_C C \\
    p_D (\frac{1}{2} A + \frac{1}{3} B) &= (\frac{1}{2} p_B) D = p_D D
\end{align*}
\]

Then:

\[
egin{align*}
    p_A &= p_C + \frac{1}{2} p_D \\
    p_B &= \frac{1}{2} p_A + \frac{1}{3} p_D \\
    p_C &= \frac{1}{2} p_A + \frac{1}{2} p_B \\
    p_D &= \frac{1}{2} p_B
\end{align*}
\]

Could solve, but if system gets more complicated, this isn't doable.

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egin{align*}
    p_A &= \frac{6}{17}, \quad p_B = \frac{4}{17}, \quad p_C = \frac{5}{17}, \quad p_D = \frac{2}{17}
\end{align*}
\]
That's too hard.

Let's use the first idea, namely start somewhere and keep running the system forward in time, but let a computer do the heavy lifting.

Our situation looks more like

...etc

This, when you have a finite number of possible positions, and "transition probabilities" telling you the prob. of going from one position to another, is called a **Markov Chain**. Can be solved in...
Illinois  .0272
B & O  .0262
Tennessee  .0256
Water Works  .0254
New York  .0251

Mediterr.  .0189
Park Place  .0191
Baltic  .0193
States  .0198
Oriental  .0201
\text{col} = \\
0.0271, \ 0.0189, \ 0.0193, \ 0.0207, \ 0.0238, \ 0.0201, \ 0.0204, \ 0.0204, \ 0.0202, \ 0.0378, \ 0.0237, \ 0.0231, \ 0.0198, \ 0.0223, \ 0.0237, \ 0.0243, \ 0.0232, \ 0.0256, \ 0.0251, \ 0.0258, \ 0.0235, \ 0.0256, \ 0.0233, \ 0.0272, \ 0.0262, \ 0.0230, \ 0.0228, \ 0.0254, \ 0.0222, \ 0.0227, \ 0.0226, \ 0.0233, \ 0.0216, \ 0.0236, \ 0.0201, \ 0.0191, \ 0.0191, \ 0.0231, \ 0.0309, \ 0.0257, \ 0.0215