

Math 331: Homework 10, Due Nov 18

1. Let $G = D_6$, the dihedral group with 12 elements.
 - (a) Write out a multiplication table for G .
 - (b) Find all subgroups of G and describe these subgroups geometrically.
2. Find all subgroups of $(\mathbb{Z}_4, +)$.
3. Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2$.
4. Let G be a group and H a subset with the following property:

$$\text{If } x, y \in H \text{ then } x^{-1}y \in H.$$

Prove that H is a subgroup of G .

5. Let G be a group and H a subgroup of G . Fix $x \in G$ and define

$$xHx^{-1} = \{xyx^{-1} : y \in H\}$$

Prove that xHx^{-1} is a subgroup of G .

6. Let G be a group with generators x and y such that

$$x^6 = y^2 = e \quad \text{and} \quad xy = yx^5$$

- (a) Show that the elements

$$x^i y^j$$

with $0 \leq i \leq 5$ and $0 \leq j \leq 1$ are the distinct elements of G .

- (b) Write out a multiplication table for G .
- (c) Find all subgroups of G .