Math 331: Homework 9, Due Nov 4

1. Determine if $x^3 + 2x + 1$ and $x^4 + 3x - 3$ are congruent $\pmod{x^2 + 2x + 2}$ in $\mathbb{Q}[x]$.

2. Determine if $x^4 + x^3 + x^2 + 2$ and $x^3 + 1$ are congruent $\pmod{x^2 + 2}$ in $\mathbb{Z}_3[x]$.

3. Prove that if F is a field, $c \in F$, $f(x) \in F[x]$, then $f(x) \equiv f(c) \pmod{x - c}$.

4. Find the principal representative of each of the following

   (a) $x^3 - x + 1, \pmod{x + 2}$ in $\mathbb{Q}[x]$.

   (b) $x^7 + x + 1, \pmod{x^3 + x + 1}$ in $\mathbb{Z}_3[x]$.

   (c) $x^4 + 2x + 4, \pmod{x^2 + 1}$ in $\mathbb{Z}_5[x]$.

   (d) $x^3 - x + 1, \pmod{x + 2}$ in $\mathbb{Q}[x]$.

   (e) $x^7 + x + 1, \pmod{x^3 + x + 1}$ in $\mathbb{Z}_3[x]$.

   (f) $x^4 + 2x, \pmod{x^2 + 1}$ in $\mathbb{Q}[x]$.

   (g) $x^3 + 3x^2 - 2x + 3, \pmod{x^3 + x + 1}$ in $\mathbb{Q}[x]$.

   (h) $x^5, \pmod{x^3 + x + 1}$ in $\mathbb{Q}[x]$.

   (i) $x^6 + x^3 + 1, \pmod{x^3 + x + 1}$ in $\mathbb{Q}[x]$.

5. Find a complete set of representatives of

$$\frac{Z_2[x]}{(x^4 + x^2 + 1)}$$

6. Find a complete set of representatives and write out the multiplication table for

$$\frac{Z_2[x]}{(x^2 + x + 1)}$$