Math 331: Homework 5, Due Oct 7

1. (a) Find all polynomials in $\mathbb{Z}_3[x]$ of degree ≤ 3. Determine which are irreducible.

   (b) Determine how many different polynomials there are of degree m in $\mathbb{Z}_p[x]$ (where p is a prime).

   Solution: For there are $(p-1)$ choices for the leading coefficient and p choices for the other $m-1$ coefficients. Thus, we have $(p-1)p^m$ different polynomials.

2. Find two examples of polynomial in $\mathbb{Z}_6[x]$ that have more roots than the degree of the polynomial.

3. Use the Euclidean Algorithm to find the gcd of the polynomials

   (a) $f(x) = x^3 + x^2 - 5x - 2$ and $g(x) = x^4 - 2x^3 - x + 2$ in $\mathbb{Q}[x]$.

   (b) $f(x) = x^3 + 2x + 2$ and $g(x) = x^4 + 3x^3 + 4x + 2$ in $\mathbb{Z}_5[x]$.

   (c) $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^4 + x^2 + 1$ in $\mathbb{Z}_2[x]$.

4. Factor completely in $\mathbb{Z}_2[x]$.

   (a) $x^4 + x^2$

   (b) $x^4 + x^3 + x$

   (c) $x^4 + x^3 + x^2 + x$

   (d) $x^4 + x^3 + x^2 + x + 1$

5. Consider polynomials $f, g \in K[x]$. How is $\deg(f \circ g)$ related to $\deg f$ and $\deg g$? Prove your result.